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CONVERGENCE OF CREDIT CAPITAL MODELS

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International Association of Credit Portfolio Managers (IACPM)
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Project Consultant
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Convergence of Economic Capital Models

Introduction and Summary[1]

The International Association of Credit Portfolio Management (IACPM) has as one of its objectives the fostering of research on credit risk measurement and management and enabling the exchange of ideas to support best practices. In addition, it seeks to represent the common interests of its members before legislative and administrative bodies and institutes. Members of the International Swaps and Derivatives Association (ISDA) share objectives similar to those of IACPM, and have consistently supported research and methodologies regarding credit risk measurement and management. The development of sound credit portfolio modeling methodologies will benefit ISDA members both in their management of counterparty credit risk and in their use of credit derivatives to hedge and diversify credit exposures.

Following up on progress in developing economic capital models made over the last eight years and since a previous consortium study was completed, IACPM and ISDA have undertaken a project to explore the convergence of economic credit capital models in use by their member firms. At the same time, new regulatory capital requirements under Basel II have been promulgated. These allow firms following an advanced approach to submit their own estimates of key parameters as input into a single regulatory formula which does not depend on portfolio composition. While the Basel Committee has expressed a willingness to explore the use of bank internal models, these have not yet been adopted by the regulators.

The principle objectives of this project are two-fold:

- To provide to banks participating in this project comparisons of the capital measures generated by different credit capital models – i.e., expected loss for the portfolio, "unexpected loss" for the portfolio, and the amount of economic capital needed to support the credit risk of the portfolio at a specified confidence level. In addition to making portfolio-level comparisons and sensitivities to changes in key parameters, to explore the assessment of credit capital measures applied to individual exposures.
- To provide to external audiences, such as the Basel Committee and other interested parties with objective, verifiable, and reproducible comparisons of internal credit capital models for their use in assessing the appropriateness of using such models for regulatory purposes.

To this end, a representative portfolio of transactions was assembled with pre-specified data assumptions regarding risk characteristics. The intent was to allow for modeling differences but not to allow for data assumptions to be the controlling variables, though some modeling differences may be the result of data interpretation. Throughout this project an overriding consideration was to develop an understanding of the various data or model assumptions that would reconcile differences in capital estimates.

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[1] For information regarding this material, contact Som-Lok Leung (somlok@iacpm.org), David Mengle (dmengle@isda.org) or Michel Araten (michel.araten@jpmchase.com)

The overall conclusions from this study is that economic capital models employed by firms can for the most part be shown to converge in their estimates of portfolio-level capital requirements, given the same data assumptions. Where differences arise, a road map of the modeling assumptions can be used to reconcile these differences. Sensitivity analysis of portfolio composition indicates that implementation of correlation assumptions may still contribute to some dispersion of results. As expected, while firm-level capital estimates have been shown to converge, how firms choose to allocate capital to individual transactions indicates significant dispersion, reflecting both the diverse purposes to which these estimates are used as well as the risk management practices specific to individual firms.

Project Participants and Governance

Members of IACPM and ISDA were invited to participate in this study. The 28 financial institutions electing to participate are listed below with Steering Committee Members in bold.

ABN AMRO	Commerzbank	Royal Bank of Canada
ANZ	Commonwealth Bank of Australia	ScotiaBank
Banca Intesa	**Credit Suisse First Boston**	Societe Generale
Bank of America	**Deutsche Bank**	SunTrust
Bank of Montreal	**Dresdner Kleinwort Wasserstein**	TD Securities
Bank of Tokyo-Mitsubishi	DZ Bank	**Wachovia**
Barclay's Capital	**JPMorgan Chase**	West LB
BNP Paribas	Key Bank	**Westpac Banking Group**
Calyon	National Australia Bank	
CIBC	PNC Financial Group	

Mich Araten from JPMorgan Chase chaired the Steering Committee with Gene Guill, Deutsche Bank and Chairman of IACPM and David Mengle, Head of Research, ISDA ex-officio. Charles Smithson of Rutter Associates provided consulting and assurance of confidentiality of data submissions.

Credit Capital Models Examined

An initial survey of the participants identified the types of credit capital models employed. The three vended models were Moody's KMV *PortfolioManager* (*PM*), RiskMetrics Group *CreditManager* (*CM*), and Credit Suisse First Boston *CreditRisk+* (*CR+*). In addition to the vended models, project participants used variations from vended models or used internally-developed models. The techniques used by the 28 participants in this study can be subdivided as follows:

- 12 obtain their official economic capital measure directly from one of the vended models
- 6 obtain their official economic capital measure from an internal model that uses the output from one of the vended models
- 8 obtain their official economic capital measure from an internally-developed model that is similar to a vended model
- 2 obtain their official economic capital measure from an internally-developed model that is significantly different from the vended models

Test Portfolio[2]

The $100 billion test portfolio is comprised of two term loans to each of 3,000 obligors across a diverse set of industries (643 NAICS codes) and 7 countries dispersed along 8 whole-grade rating buckets and varying LGDs. Exposure amounts varied from $1MM to $1,250MM and tenors ranged from 6 months to 7 years. "R-squares" (the degree to which obligors exhibit systematic vs. idiosyncratic risk) varied from 10% to 65%. Contractual spreads over a risk free rate were chosen so that the market to market value of the exposures at time zero relative to specified required market spreads, would be approximately par. The characteristics of the test portfolio are provided in Exhibit 1.

Exhibit 1
Characteristics of Test Portfolio

Portfolio Size	
Exposures	6000
Portfolio Size	$100 Billion
Obligors	
Number of Obligors	3000
Rating Scheme	8 ratings buckets
Credit rating	Average = BBB
Industry Classifications	61 M-KMV industries *643 NAICS Codes (6 digit)*
Countries	7 countries
Facilities (Term Loans)	
LGD	22% to 58% Average = 40.6%
Fixed vs Floating	100% Floating Rate
Exposure Distribution by Facility	
Mean	$16.7 million
Standard Deviation	$101.7 million
Minimum	$1 million
Maximum	$1,250 million
Tenor Distribution by Facility	
Mean	2.5 years
Standard Deviation	1.7 years
Minimum	6 months
Maximum	7 years
Correlation	
R-squared	Average = 20%

Project Phases

- Phase 1 of the project was the analysis of key results of running the portfolio models in two modes -- "default mode" and "mark-to-market mode" using a set of standardized and pre-specified base settings. Participants were also able to submit results of their models in "production" modes allowing variation in model settings

[2] A zipped file containing the data set and assumptions is available upon request addressed to Som-Lok Leung (somlok@iacpm.org) or David Mengle (dmengle@isda.org)

but not in data assumptions. For comparative purposes, required capital under Basel II was also determined.

- Phase 2 consisted of exploring changes to key data assumptions and related sensitivities of portfolio capital to explore how models reacted to data assumptions. Basel II capital was also recalculated based on the changed input data.

- Phase 3 consisted of comparing attributed capital to a small set of individual transactions. It was felt that while credit capital models might be shown to converge at the portfolio level, bank practices associated with allocating capital to individual exposures were likely to be highly varied.

Analysis Phase 1

The objective of Phase 1 was to compare the capital measures (at the portfolio level) generated by different credit capital models – i.e., expected loss for the portfolio, "unexpected loss" for the portfolio, and the amount of economic capital needed to support the credit risk of the portfolio at various specified confidence levels.

All 28 of the project participants submitted Phase 1 responses (with one of the 28 submitting responses based on two different credit capital models). The responses were based on 7 different types of credit capital models.

The initial inspection of the "Default Only" mode responses submitted by project participants suggested that the results from *PM* and similar models while consistent within models were very different from the results from *CM* and *CR+* and similar models. (Exhibit 2) The average Expected Loss and the average Economic Capital at the 99.90% confidence level generated by *PM* and similar models were significantly larger than those generated by *CM* and *CR+* and similar models

(Exhibit 2 Default Only Base Runs)

Default Only ($MM)	Expected Loss		Capital at 99.90%	
	Mean	Std Dev/Mean	Mean	Std Dev/Mean
PM and Similar Models	789.5	3%	4,419.5	4%
CM and Similar Models	565.5	2%	3,816.7	10%
CR+ and Similar Models	563.8	0%	3,387.3	10%
Basel II (with caps/floors—min 1 yr maturity and .03 bps PD)	606.8		3,345.2	

However, it turned out that this difference results from the way the models treat the four quarterly interest payments in event of a simulated default at the one-year horizon. In the case of *PM*, all of the coupons that were owed between time zero and the horizon are included in the loss in the case of default at the horizon. *CM* and *CR+* effectively presume that the obligor pays all of the coupons. In this instance the loss is measured with respect to the principal, and excludes the coupons between initial and horizon date.

To demonstrate this, *PM* was rerun, with spreads and the risk free rate set equal to zero. The results of this demonstration created as Working Group (WG) runs (Exhibit 3) illustrated that, when spreads and the risk free rate are set to zero, *PM* produces results that are very similar to those produced by *CM* and *CR+*. The implication is that, if there are no coupons subject to loss in the "Default Only" mode, the three models converge.

Exhibit 3 (Default Only)

Default Only ($MM)	Expected Loss	Capital at 99.90%
WG Run of PM with spreads and risk free rate set to zero	563.4	3,791.2
WG Run of CM	561.6	3,533.2
WG Run of CR+	563.8	3,662.0

That the difference in treatment of coupons in the definition of loss between the different types of models leads to such large differences in outcomes can be explained by the observation that expected loss, and to a lesser extent economic capital, are largely determined by the bad-quality credits in the portfolio. For these credits, the coupons were set quite high in order to let them price to par at the initial date.

Initial inspection of the "Mark to Market" provided results similar to those of the initial examination of the "Default Only" mode responses (Exhibit 4): The average Economic Capital at the 99.90% confidence level generated by *PM* and similar models were significantly larger than those generated by *CM* and similar models. (*CR+* as a default only model was not included.)

Exhibit 4 (Mark to Market Base Runs)

Mark-to-Market ($MM)	Expected Loss		Capital at 99.90%	
	Mean	Std Dev/Mean	Mean	Std Dev/Mean
PM and Similar Models	789.5	3%	5,617.5	7%
CM and Similar Models	761.3	23%	4,823.4	10%
Basel II (with caps/floors—min 1 yr maturity and .03 bps PD)	606.8		4,208.4	

These differences led the project group to carefully examine analytical similarities and differences in the two models:

- Valuation methodologies: If assumptions are aligned, there is not much difference between the valuation methods from *PM* and *CM*. In practice, the differences that exist are due to the different default claim assumption
- Expected Loss: As noted above, "Expected Loss" in *PM* includes all coupons, while "Expected Loss from Horizon Value" in *CM* includes only interest that is accrued but not yet paid.
- Application of LGD: In *PM*, the LGDs are implicitly assumed to have been derived from determining the exposure at default (EAD) and identifying all cash flows received (positive or negative and regardless of whether they represent principal or interest or fees paid out), discounting them at some rate and calculating the present value of the cash flows relative to the EAD. In practice, some banks will add to the EAD any unpaid interest, while others will only

capture the principal amount outstanding, but *PM* assumes the LGD incorporates all unpaid interest. In *CM*, consistent with the description of "Expected Loss from Horizon Value" above, recovery claims in the event of default equal the principal of the bond plus accrued but unpaid interest.

- Point in Time to which Economic Capital Applies: In *PM*, economic capital is reported at time zero. The capital distribution takes losses incurred/measured at the horizon and discounts them back to time zero at the risk free discount rate. In contrast, in *CM*, economic capital is reported at the horizon. Using the same valuation point would decrease *CM's* reported capital by approximately 5.1%.

To determine the degree to which LGD assumptions associated with coupons versus correlation assumptions accounted for the differences in economic capital, the portfolio was simplified by setting the maturities of all transactions to one year, thereby controlling for differences in market valuation methodologies between models. Alternative credit capital results were obtained changing (1) the spreads and the risk-free rate and (2) the degree of diversification of the portfolio (Exhibit 5).

Exhibit 5 (Default Only Runs)

Default Mode	Run 1		Run 2		Run 3		Run 4	
Spreads, Coupons, RiskFree Rate	On		On		Off		Off	
Industry	All		"Unassigned"		All		"Unassigned"	
Country	All		US		All		US	
Model	CM	PM	CM	PM	CM	PM	CM	PM
Expected Loss	567.8	862.7	567.8	862.7	605.4	605.2	605.4	605.2
Capital (99.9%)	3,467.1	4,626.3	6,633.1	8,331.7	3,756.8	3,890.0	7,221.5	7,148.1

Again, we see in run 3 that when we eliminate all spreads, coupons, and the risk free rate we obtain capital convergence. We also see convergence in run 4 even where in addition to eliminating spreads, coupons, and risk free rate, all exposures were assumed to be in a single unassigned industry within a single country. However, when the same contrasting portfolio composition assumptions were tested in runs 1 and 2, but spreads, coupons, and the risk free rate were not eliminated, we see significant capital divergence. We can thus assume that the differences associated with LGD assumptions as found in default mode or when maturities are set to one year will carry forward in a mark-to-market mode with a full range of maturities. This empirical analysis suggests that differences in the modeling of correlation have a smaller impact than do differences in the ways the models treat interest payments.

However, in a full "Mark-to-Market" mode, where changes in revaluations at the horizon for non-defaulted assets may also be correlated, the impact of differences in the modeling of correlations and other modeling may have a larger impact.

In the same manner as in Default Only mode, runs were made to isolate the effects of the LGD assumptions discussed above, correlations, and other modeling differences. In this analysis, as can be seen in Exhibit 6, it was determined that the LGD assumptions and correlation assumptions each contribute about 25% to the differences in model results, and modeling differences account for approximately 50% of the difference. While initially Mark-to-Market capital differences in the base runs were as high as 25%, by

controlling for LGD assumptions that impact coupons and by creating a simpler set of correlation assumptions, the capital differences were reduced to about 12%.Thus, while assumptions can be aligned as far as LGDs and correlations, one is still left with different modeling methodologies that may not be so completely aligned.

Exhibit 6 (Mark to Market)

Mark to Market Mode	Run 1		Run 2		Run 3		Run 4	
Spreads, Coupons, RiskFree Rate	On		On		Off		Off	
Industry	All		"Unassigned"		All		"Unassigned"	
Country	All		US		All		US	
Model	CM	PM	CM	PM	CM	PM	CM	PM
Expected Loss	728.0	793.3	728.0	793.3	561.6	418.1	561.6	418.1
Capital (99.9%)	4,182.3	5,600.0	7,946.0	9,794.0	3,501.6	4,321.5	6,802.2	7,749.3

Analysis Phase 2

The objective of Phase 2 was to compare and contrast the sensitivities of the various credit capital models to changes in concentrations and parameters.

The following sensitivities were examined:

- Separating investment and non-investment grade transactions into two separate portfolios, summing their separate capital and comparing this sum to the single combined portfolio's capital
- Concentration changes
  - Change in geography and/or industry concentration
  - Change in individual exposure concentration
- Changes in parameters
  - Changes in probability of default
  - Changes in market spreads
  - Change in the risk free rate
  - Change in loss given default (LGD)
  - Changes in tenor
  - Changes in R-squared (*PM*-type and *CM*-type models) or the volatility of the probability of default (*CR*+ type models).

A total of 16 sets of sensitivities to the changes were obtained:

- Sensitivities of *PM* and *CM* obtained by the working group
- Sensitivities submitted by 13 project participants
- Sensitivities based on Basel II pro forma calculations

Exhibit 7
Sensitivities: Changes in Portfolio Capital from Base Levels

Scenario		Average All Models		
		Default Mode	MTM Mode	Proforma Basel II
Segment Portfolio into Investment and Non-Investment-Grade Sub-Portfolios		12.9%	9.3%	0.0%
Geog. Concentration	Change all countries to "US"	33.0%	32.8%	0.0%
Industry Concentration	Change all industries to "Unassigned"	17.2%	18.6%	0.0%
	Change all industries to "Telecom"	25.3%	25.9%	0.0%
Geog. and Industry	Change countries to "US" /industries to "Unassigned"	77.2%	78.3%	0.0%
Individual Exposure Concentration	Increase 1 telecom exposure from 5MM to 1,005MM	0.1%	0.3%	0.4%
	Increase 1 telecom exposure from 5MM to 5,005MM	2.3%	5.0%	2.2%
	Increase 1 broadcast exposure from 500MM to 1,500MM	1.2%	1.4%	1.2%
	Increase 1 broadcast exposure from 500MM to 5,500MM	19.2%	17.6%	6.2%
Prob of Default (PD)	Increase all Prob of Default by 20%	7.6%	6.3%	6.4%
	Increase all Prob of Default by 40%	14.1%	12.0%	11.9%
Market Spreads	Increase Sharpe Ratio to 0.5 -OR- use Alt Spread #1	0.0%	0.3%	0.0%
	Increase Sharpe Ratio to 0.6 -OR- use Alt Spread #2	0.0%	0.7%	0.0%
Risk Free Rate	Reduce to 2%	-1.4%	-0.8%	0.0%
LGD	Increase all by 20%	17.3%	14.8%	20.0%
Tenor	Increase maturities on all exposures by 1 year	NA	12.9%	11.1%
	Increase maturities on all exposures by 2 years	NA	20.9%	23.1%
R-Squared : Increase by 20%		14.9%	14.3%	0.0%

The results of the sensitivity tests can be summarized as follows:

- Segmenting portfolio into Investment-Grade and Non-Investment Grade sub-portfolios: In contrast to the Basel II approach which is invariant to this segmentation, the economic capital obtained from the credit capital models used by the project participants increased
- Geographic and/or Industry Concentrations: In contrast to the Basel II approach, all of the models used by participants are sensitive to changes in geography and/or industry concentrations.
  - Geographic concentration: *PM*–type models are more sensitive to this change than are the other model types.
  - Industry concentration: *PM*–type models are less sensitive to changing all industries to "Unassigned" than to changing all industries to "Telecom"; *CM*–type models exhibit the reverse pattern.
  - Combining Industry and Geographic concentration: Changing all countries to "US" and all industries to "Unassigned" has a dramatic impact on the concentration of the portfolio. Economic capital numbers obtained from *PM*-type and *CM*-type models increasing about the same amount, almost doubling.
- Individual Exposure concentration: The models used by the participants are somewhat more sensitive to the changes we examined than is the Basel II approach.
- Probability of Default: For a 20% increase in probabilities of default, the increases obtained from the models used by the project participants are

approximately the same as from the Basel II approach. For a 40% increase in probabilities of default, the increases obtained from *PM* and *CM* are somewhat larger than (in the "Default Only" mode) or approximately equal to (in the "Mark-to-Market" mode) those from the Basel II approach,.

- Market Spreads: The Basel II approach is invariant to this change and the sensitivities obtained from the models used by the project participants are effectively zero.
- Risk Free Rate: The Basel II approach is invariant to this change. *PM*-type models are more sensitive than are *CM*-type models.
- LGD: The models used by the project participants produced changes that are similar but not quite as linear as that produced using the Basel II approach.
- R-squared: The Basel II approach is invariant to this change; all of the models used by project participants produced similar increases in economic capital.
- Tenor ("Mark-to-Market" mode only): For a 1-year increase in tenor, both *PM*-type and *CM*-type models produce increases in economic capital that are slightly larger than that obtained using the Basel II approach. For a 2-year increase in tenor, *PM*-type models produce increases in economic capital slightly smaller than that obtained using the Basel II approach, while the increases produced by *CM*-type models are approximately the same as that obtained using the Basel II approach.

The Phase 2 results lead to the following conclusions:

1. The models that all participants in the study use for their internal economic capital estimates are sensitive to changes in portfolio concentration. Such changes may arise from a change in the number of exposures in the portfolio, changes in geography and/or industry, or changes in the size of individual exposures. The results show that the effect of changes in concentration on economic capital can be very significant. This contrasts with the Basel II regulatory capital calculations, which are insensitive to changes in portfolio concentration.
2. The size of economic capital changes as a result of changes in portfolio concentration can differ significantly between different models. These differences are generally largest between different types of models, and smaller between models that belong to the same class. The observed differences in concentration effects between models may point to differences in correlation between the various models. Such differences in correlation can be structural in nature, as different participants may use different data to calibrate correlations (e.g., historical equity returns versus default rate data). Even if participants use the same type of data (e.g., equity returns), calibrated correlations may differ if different historical time periods have been used in the calibration, as it has been observed that correlations vary over time.
3. Most of the models that are used by the participants react quite similarly to changes in the input parameters PD, LGD, R-squared, risk-free rate, and market spreads. The observed differences with the sensitivities of the Basel II regulatory capital calculations can for some parameters be attributed to the fact that they are not a direct input into the Basel II formula (risk-free rate, market spreads, and R-squared). The project has not explored further the reasons for the difference in

impact of the changes in PD and LGD, but these may be due to the role that portfolio concentration effects play in the internal models, and/or the fact that internal models include uncertainty in the LGD while Basel II does not.

Analysis Phase 3

The objective of Phase 3 was to compare the risk contributions that different participants would assign to specific transactions or the transactions of a defined cohort of obligors.

Project participants were asked to provide information about 8 individual transactions and two cohorts of obligors.

In order to compare the risk contributions, project participants were asked to characterize their risk contributions:

- Mode: "Default Only" or "Mark-to-Market"
- Type of Risk Contribution: "Standard-deviation-based" risk contribution or "Tail-based" risk contribution
- Use of risk contribution: Performance measurement or Pricing

Twenty three Phase 3 submissions were received. The submissions suggest that there is very little consistency with respect to the risk contributions calculated by individual institutions. Within each of the groups of model types, there are many variants that could be used for either standard-deviation-based or tail-based risk contributions. How firms choose to allocate capital to individual transactions indicates significant dispersion, reflecting both the diverse purposes to which these estimates are used as well as the risk management practices specific to individual firms.

Conclusions

Credit Risk modeling practices at major international financial institutions has been explored using a fixed set of data inputs. Across both vended and internal models it can be shown that when loss assumptions are aligned, estimates of credit capital can be shown to converge at least in Default Only models. Differences in Mark-to-Market models can also be reduced, but not eliminated completely. Sensitivities to changes in parameters produce somewhat similar and expected changes in credit capital and stand in contrast to Basel II calculations which are not sensitive to portfolio composition. Finally, as expected, there is a rather diverse set of internal practices when it comes to allocating overall portfolio capital to individual transactions in the portfolio.